Abacaba-Dabacaba!

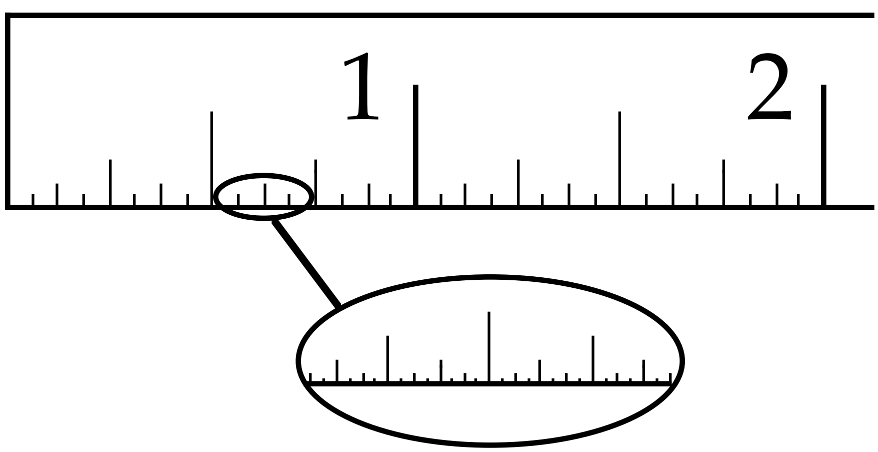
by Michael Naylor
Western Washington University

The Abacaba structure shows up in an amazing variety of places. This article explores 10 surprising ideas which all share this pattern, a path that will take us through geometry, number systems, art, music, poetry, higher dimensions, and more!

Did I say 10 places? Actually, this one goes to 11.

1. Ruler

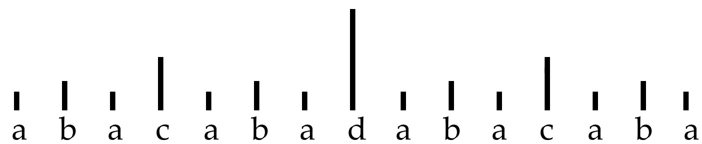
Take a look at the marks on ruler in the space of one inch. There's a big mark dividing the inch in half, two shorter marks dividing those halves in half, and so on. It's easy to imagine you could keep dividing the ruler again and again infinitely. If you did, you'd have a very interesting geometric object called a fractal.



If the shortest marks are length 1, the next length 2, and so on, the pattern of the marks is 1213121412131215... This pattern shows up in all kinds of weird places; let's give it a name.

2. The Name

Instead of using numbers to describe the length of branches or marks on the ruler, let's call the shortest lengths "a", the next longest "b," then "c," and so on. The pattern then becomes:



. . . or "Abacaba-Dabacaba!" This word sounds very much like the magician's phrase "abracadabra," and indeed there are seemingly magical properties about this pattern.

To understand the pattern a little better and see how to continue it, let's see how this pattern grows. Start with an "a."

1. **a**

To grow the pattern, add the next letter in the alphabet and then repeat everything that has gone before (which is just the letter "a" in this case.) The next step, then, is "aba."

2. **aba**

Continue by adding the next letter, "c," and repeating the "aba."

3. **abacaba**

The fourth step adds the letter "d" and repeats the pattern: abacabadabacaba! The next few steps are shown:

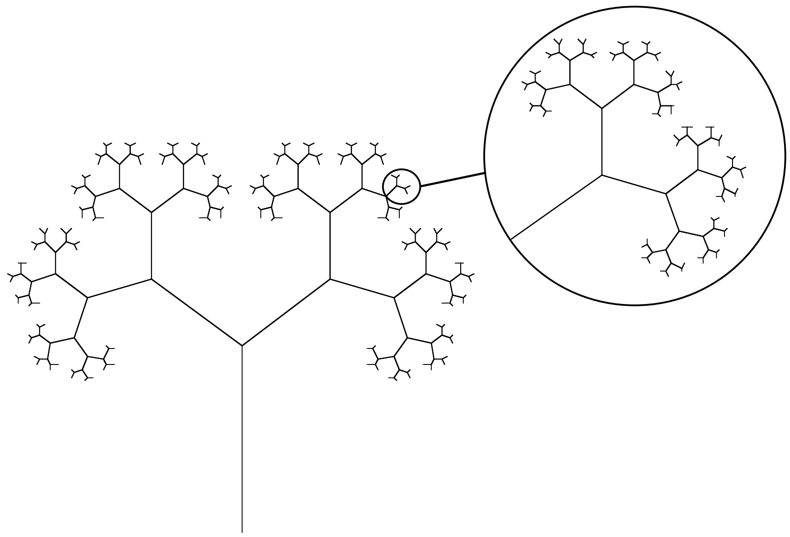
4. **abacabadabacaba**
5. **abacabadabacabaeabacabadabacaba**
6. **abacabadabacabaeabacabadabacaba-fabacabadabacabaeabacabadabacaba**

It's fun to see how much you can say aloud. How long would it take to say the word all the way to "z"?

This word is the basis for genie names in *Maggie and the Abacaba Genies* (see www.abacabax.com). In the story, Maggie must call forth genies by saying these names, all the way to z!

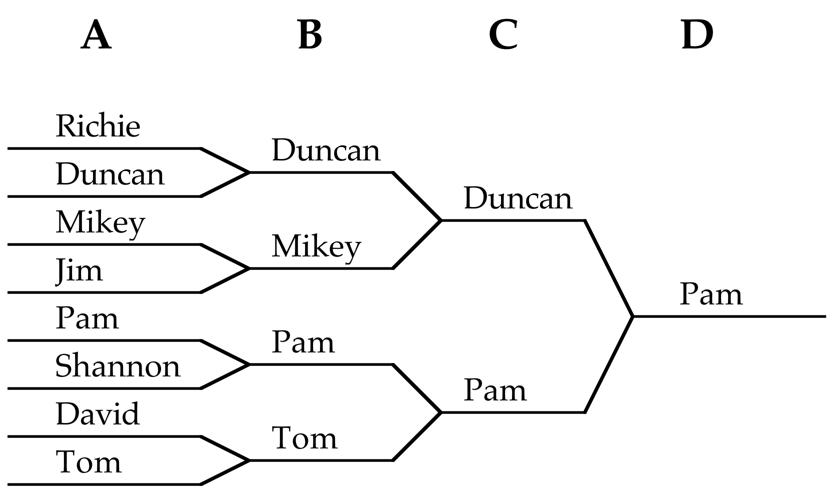
3. The Tree

Another way to represent an Abacaba patterns is with a binary branching tree. In this pattern, one object divides into two, then each of those divides into two, and so on, like the branches on this tree:



It is not hard to imagine that the branching and doubling could be continued infinitely, and if we could only magnify our view enough times, we would see the same patterns continuing forever and ever...

This same pattern is followed (in reverse) in a play-off schedule where teams or players are paired off with the winner of each round progressing to the next while the loser is eliminated:



If we move from the top to the bottom of the playoff tree, we'll notice that the name closest to the top of the chart appears in column A, the next highest name is in column B, then column A, then C, then back to A, and so on. The pattern is abacaba-dabacaba.

4. Fractals

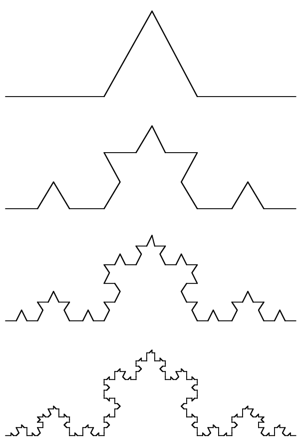
Abacabadabacaba is a fractal pattern – and it shows up in many other fractal patterns. Fractals, like the ruler and the tree, have parts that are similar to the whole – if you zoom in and look closer, you see the patterns over and over. Here are three more famous fractals.

The **Koch Curve** is formed by replacing the center third of a line segment with two edges of an equilateral triangle.

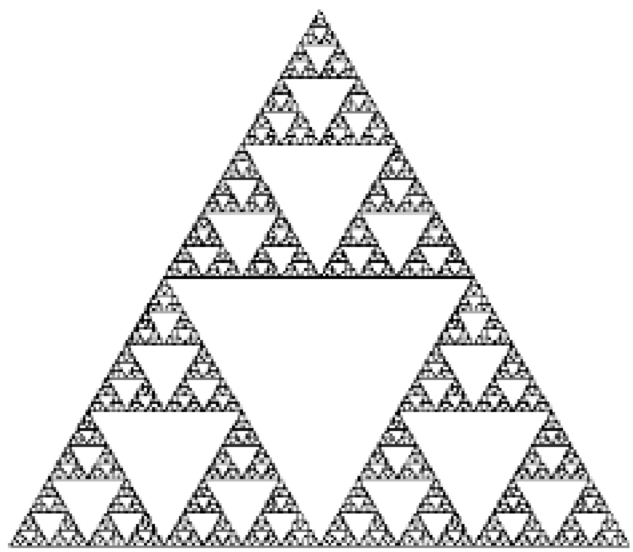
The **Sierpinski Triangle** (or Gasket) is formed by removing the center of a triangle and repeating

Cantor Dust is made by successively removing the center third of a line segment.

Can you find abacabadabacaba in each of these?



Koch Curve



Sierpinski Triangle

Cantor Dust

5. Number Systems

The binary number system is the simplest and most significant of all positional number systems.

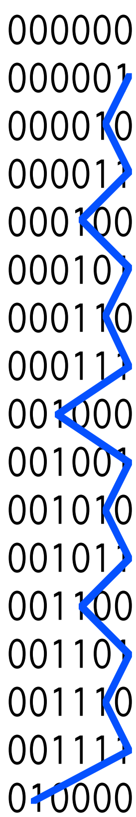
In binary, only two digits are used, zeroes and ones. The value of a written number is determined by the arrangement of zeros and ones, with the value of each place being a power of 2 rather than a power of 10 as in our familiar system. The number 19, for example, would be written as follows:

19 (base 10)	=	**0**	**1**	**0**	**0**	**1**	**1**	digits of binary #
		32	*16*	*8*	*4*	*2*	*1*	value of each digit
		2^5	2^4	2^3	2^2	2^1	2^0	power of 2

$$19 = 2^4 + 2^1 + 2^0$$

The count from 0 to 8 in binary is: 0, 1, 10, 11, 100, 101, 110, 111, 1000

The abacabadabacaba pattern is repeated at infinite levels in this simple counting pattern. The start of one such pattern is shown to the left. Can you find others?



0	000000
1	000001
2	000010
3	000011
4	000100
5	000101
6	000110
7	000111
8	001000
9	001001
10	001010
11	001011
12	001100
13	001101
14	001110
15	001111
16	010000
17	010001
18	010010
19	010011
20	010100
21	010101
22	010110
23	010111
24	011000
25	011001
26	011010
27	011011
28	011100
29	011101
30	011110
31	011111
32	100000

6. Philosophy and Poetry

It's fun to think about how every decision we make leads us in a new direction, as if our lives are an infinite fractal tree. Here's a poem that reflects those decisions... it has the structure *abacabadabacaba*.

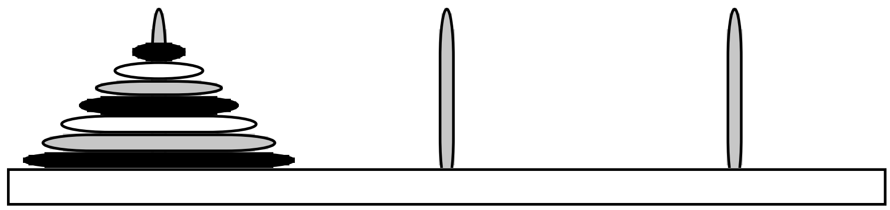
And keep my conscience clear and bright.
I'll do what I know is right
Next time, who knows? I just might!
"I shouldn't do it," so I thought.
I guess I'm doomed to live this way.
Now the chance has slipped away
Maybe on some other day!
Should I do it? Should I not?
It wasn't worth it, I would say.
Now I'm full of guilty thoughts
But that's a tiny price to pay!
I went and did it anyway.
Tomorrow I will stay away.
I'll just hope I don't get caught
They didn't catch me yesterday!

Decision Tree

by Michael Naylor

(*College Math Journal*, 32, 3, May 2001.)

7. Puzzles and Legends

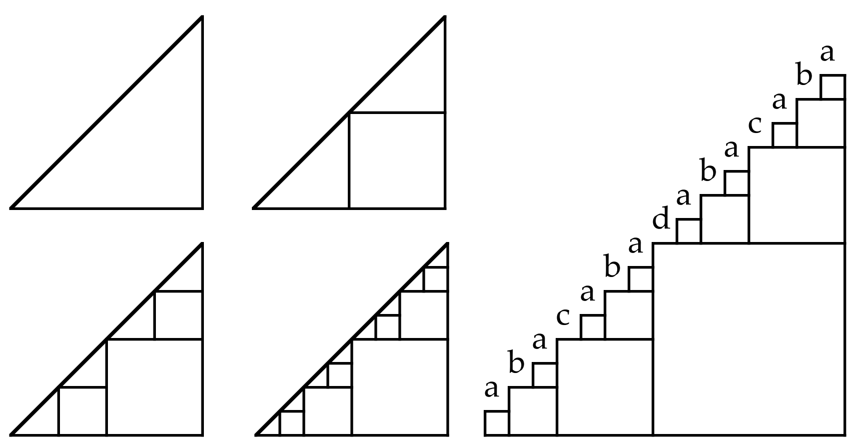


A popular puzzle called "The Towers of Hanoi" asks players to move a stack of disks one at a time from one of three pegs to another peg. Only the top disc of a tower may be moved, and one may never place a larger disc on top of a smaller disc. The object is to transfer the entire tower to a different peg in the fewest number of moves.

A version of this game using cards is at the end of this article. The key to solving this puzzle lies in the abacaba pattern.

There is a legend of the Temple of Abacabax, where monks move golden disks on diamond spindles. When they have moved all 26, the universe will come to an end. Should we worry?

The Temple of Abacabax is high atop of tower of stone blocks. The blocks are arranged as such: Draw half a square, cut along the diagonal to form a right isosceles triangle, and then draw the largest square possible inside of it. Continue by placing the largest square possible inside of all the right triangles created, and repeat. When you decide to stop, you will have made a staircase of blocks similar to those at the temple of Abacabax. As you climb these stairs, the size of the blocks you step on makes the abacabadabacaba pattern. In the staircase at the actual temple, this process is carried out 26 times, the central block corresponding to the letter 'z.'

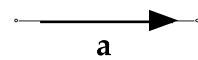


At the end of this article you'll also find a template to make a pop-up version of this staircase.

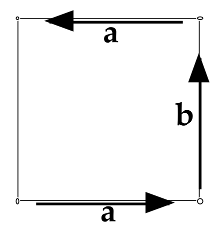
8. Hyperspace

Navigating higher dimensions? Don’t get lost! Call the *left-right* direction “**a**,” the *up-down* direction “**b**,” and the *forward-back* direction “**c**.”

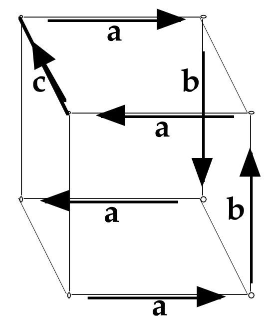
Moving **a** will get you from one point to another – you travel a line segment.



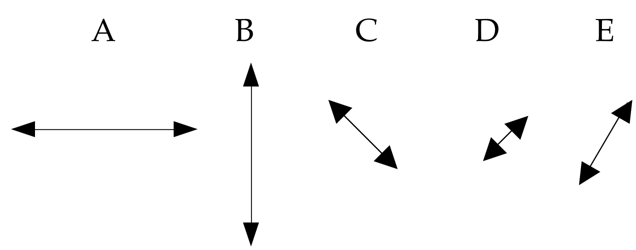
Moving **aba** will move you around the corners of a square.



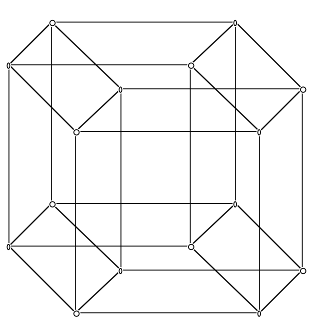
Try moving **abacaba** on the cube - did you visit all of its vertices?

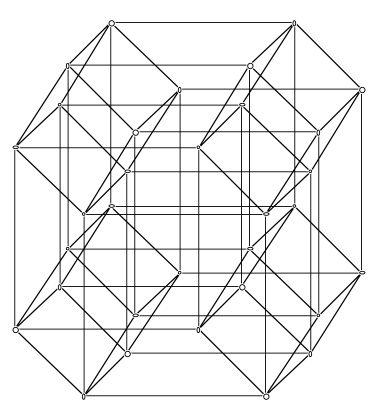


Let’s add another direction, the *here-there* direction, and call it **d**. To travel to all of the vertices of this four dimensional hypercube, just remember the magic word: Abacabadabacaba!



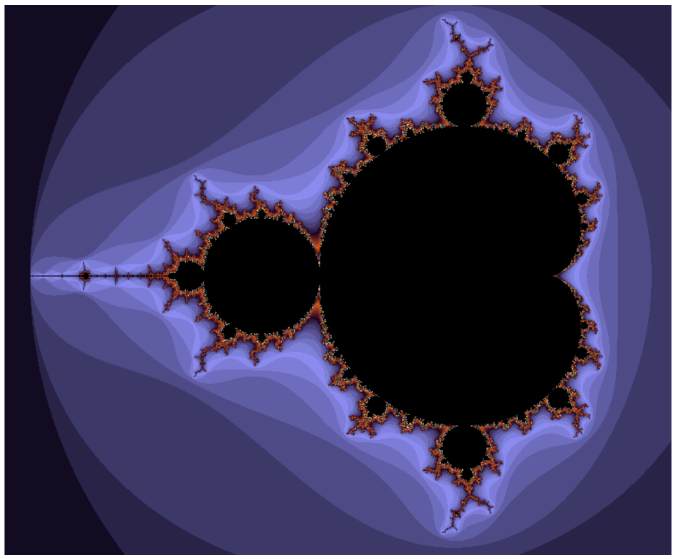
Here’s a 4D and a 5D hypercube. Are you up to the challenge?



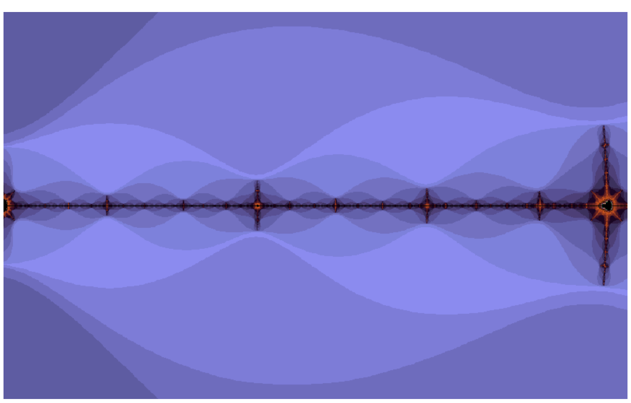


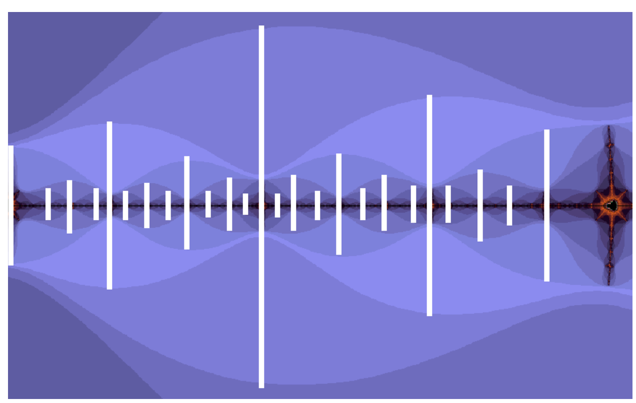
9. The Complex Plane

Abacaba-Dabacaba is written all over the Mandelbrot set. The M-Set is a famous fractal with infinite variety and complexity, generated by repeating very simple rules. Many free computer programs are available to explore this fantastic structures. (These were made with the freeware program *Xaos*).



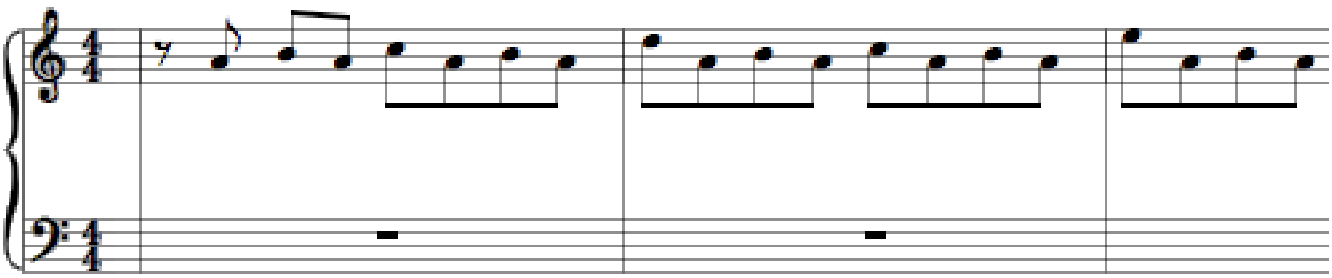
Zooming in on the “nose” to the far left reveals one abacaba pattern as shown here – there are many more you can find.





10. Music

Abacabadabacaba sounds like it could represent a series of notes. Indeed, if abacabadabacaba is played on piano, the result is beautiful and haunting. Download a copy from **abacabax.com**. There is also a fully orchestrated version available for free download.



11. www.Abacabax.com

More Abacaba available online for FREE

Read *Maggie and the Abacaba Genies*

Download Classroom Activities, Worksheets, and Blackline Masters

Download Abacaba Music

Cheers!

mike@abacabax.com

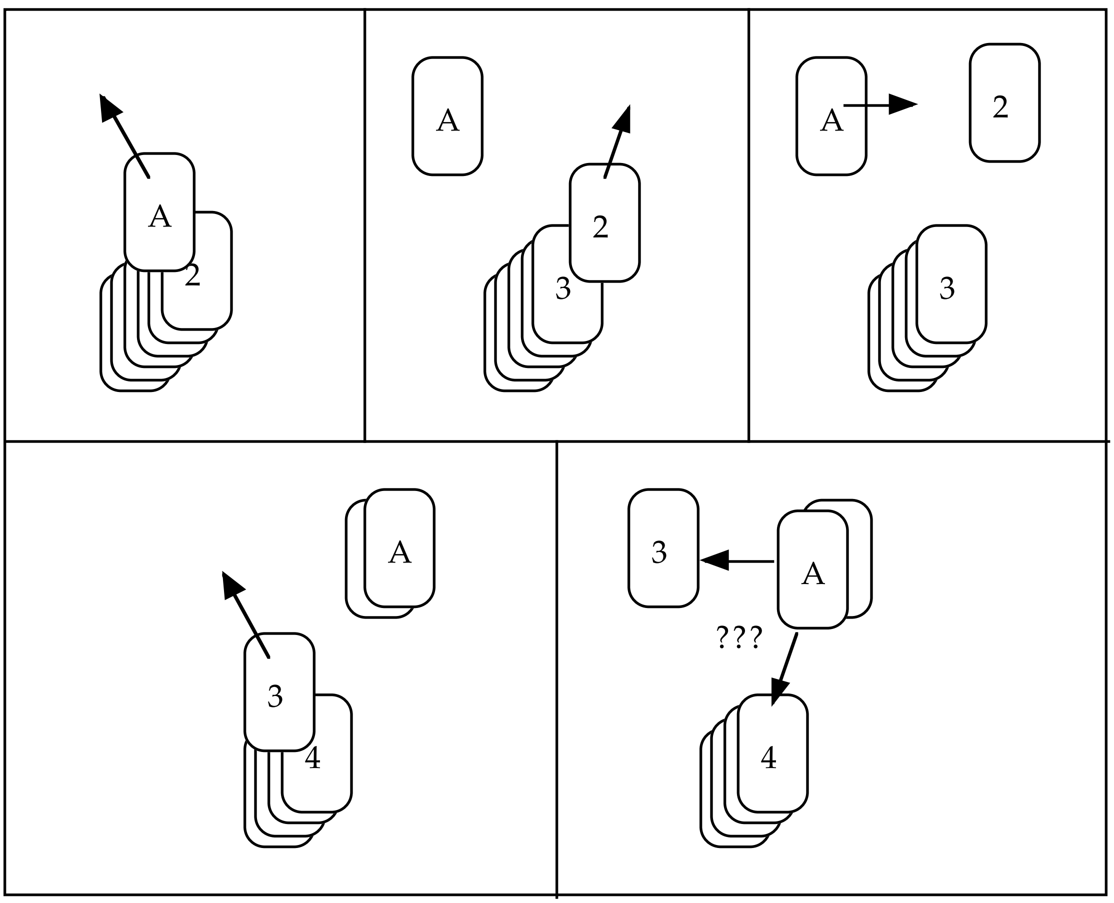
Abacabax

Roly-Annie

"Roly-Annie" is a solitaire card game named after the card queen of Mississippi riverboats, "Roly" Annie Keim. To play, use the cards ace through seven. Make the odd numbered cards black and the even numbered cards red, and stack them in order so that when the stack is face up, the ace is on top.

You are allowed to make two additional stacks in this game, a right stack and a left stack. You're allowed to move **the top card** of any stack (on the table or in your hand) to the top of any other stack (on the table or in your hand). However, you may move **only one card** at a time, and you may never place a higher numbered card on top of a lower numbered card; you may **only place a lower number on a higher number** (the ace has a value of 1). You may play any card on an empty stack.

The **object** of the game is get all of the cards into either the left or right stack. The start of a game is shown below.



1. What is the minimum number of moves required to win this game?
2. List at least the first 20 moves. (You may wish to devise your own notation.)
3. Describe any patterns you found while investigating this game. Describe a strategy that will allow you to win every time without making a mistake.
4. You and a friend decide to race. You split a deck of cards and number the cards in each half from 1 to 26. Playing by the same rules as before, about how long would it take to decide a winner?

POP-UP FRACTAL

This eye-popping fractal is a big hit with kids (and grown-ups, too)! It's easy to create something complicated and beautiful using ideas of fractals. The following page contains the pattern for making this work of art.

Supplies:

> Copies of the handout on the next page
> Scissors
> optional: 9" x 12" construction paper and glue sticks

1. Have students fold the page in half along the dotted line. Fold so that the print is on the outside of the folded page.

2. Cut along the two heavy line segments that connect to the crease and fold the flap upwards. The corners of the flap will fit neatly in the "L" brackets printed on the page.

3. Open the page, reverse the fold in the center of the flap and close the page so the flap is now completely (and neatly!) inside the folded paper.

4. Two move heavy lines are now seen to reach the creases in the center of the paper. Cut along these also, then crease, using the "L" brackets to help. After folding the flaps up, you can flip the paper over and fold the flaps in the opposite direction. This will make it easier to reverse the flaps and pop them inside.

5. When you've popped the two smaller flaps inside, you should now be ready to make two more cuts along the heavy lines that now meet the four creases. Continue in this manner.

> *Point out that the first cuts made one flap, the second cut made two flaps, and the third made 4. How many flaps are made by the final cut?*

When it is done, you will have a beautiful 3-D pop-up form. This can be glued to a folded sheet of construction paper to make a lovely piece of art or an eye-popping greeting card.

> *Questions for discussion:*
>
> > *How is this a fractal? How is this self-similar?*
> > *How many of each size block are there?*
> > *What other properties can you investigate?*
> > *What do you notice when you look at your model from different perspectives?*