Teaching Materials to Accompany

"Maggie and the Abacaba Genies"

Grades 3-8

Additional Support Materials available at:

www.abacabadabacaba.net

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ALGEBRAIC PATTERNS in ABACABA

Teacher Guide to MAGGIE AND THE ABACABA GENIES – PATTERNS 1

This is a great first activity after reading "Magging and the Abacaba Genies" to your class. Give your students copies of the worksheet and demonstrate on the board how the genie names are generated using the rules on the top of the page.

Step 1: A

Step 2: Now write B after the A and repeat everything that came before (in this case, just the letter A). This makes: ABA

Step 3: Now write C after the ABA and repeat the ABA. This makes ABA-C-ABA.

Step 4: This next step adds the letter D and repeats the ABACABA to make ABACABA-D-ABACABA.

Point out that this is the name of the first genie we meet in the book, but it is actually the 4th step in the process. When this genie introduces herself, she says she is the "ruler of the 4th dimension, guardian of the Depths." Her name is the 4th step in this process, so she rules the 4th dimension. Notice also that her name goes to the letter D, and she guards the Depths, which also begins with D.

Now have your students work to complete the chart and answer the questions on the back. It is best to discuss questions 1-4 before having your students work on question 5, or assign question 5 as homework.

ANSWER KEY

1. The completed chart and ***some*** of the patterns in the chart are shown on the next page. (There are more!) If your students have worked with exponents, be sure to bring out the idea of "powers of 2," and that *2 to the power of the genie # minus 1* = $2^g - 1$ = *# letters in name*. This is a powerful shortcut to the later questions!
2. The 10th genie has 1023 letters in her name.
3. The 20th genie has 1,048,575 letters in her name.
4. The 26th genie has 67,108,865 letters in her name!
5. Shortcut: $2^g \div 8$ = # seconds, or 2^{g-3} = # seconds.

Number of seconds:	8,388,608
Number of minutes:	~139,810
Number of hours:	~2,330
Number of days:	~97

That's more than 3 months... non-stop 24 hours per day!

6. Is it possible? Not by a human in one sitting!

Extension: How long would it take to say the name if you took breaks for eating, sleeping, etc.?

Teacher Guide for MAGGIE AND THE ABACABA GENIES – PATTERNS 1, continued

Genie #	Name?	How much longer is this name than the previous genie's?	Total Length of this name?
(1)	A	1	1
(2)	ABA	2	3
(3)	ABACABA	4	7
4	ABACABADABACABA	8	15
5	ABACABADABACABA– EABACABADABACABA	16	31
6	ABACABADABACABAEABACABADABACABA- FABACABADABACABAEABACABADABACABA	32	63
7	ABACABADABACABAEABACABADABACABA FABACABADABACABAEABACABADABACABA GABACABADABACABAEABACABADABACABA FABACABADABACABAEABACABADABACABA	64	127
8	ABACABADABACABAEABACABADABACABAFABACABADABACABAEABACABADABACABA GABACABADABACABAEABACABADABACABAFABACABADABACABAEABACABADABACABA HABACABADABACABAEABACABADABACABAFABACABADABACABAEABACABADABACABA GABACABADABACABAEABACABADABACABAFABACABADABACABAEABACABADABACABA	128	255
9	- - -	256	511

difference of 1

difference of 1

times 2, plus 1

times 2, plus 1

These numbers double and are powers of 2

These numbers are one less than powers of 2

The total length of the genie's name is 2 to the power of the genie # minus 1: $2^g - 1$

Maggie and the Abacaba Genies 1 Name ______________________

RULES FOR MAKING GENIE NAMES:

Start with A. Write the genie's name **twice**.
Write the **next letter** of the alphabet between the two names.
Now you have the name of the next genie. Repeat.

Complete this chart of Genie Names:

Genie #	Name	How many letters?
(1)	A	1
(2)	A B A	3
(3)	_______ C ________	7
4	_________________ D _________________	_____
5	_________________________ E _________________________	_____
6	ABACABADABACABAEABACABADABACABA- FABACABADABACABAEABACABADABACABA	_____
7	ABACABADABACABAEABACABADABACABA FABACABADABACABAEABACABADABACABA GABACABADABACABAEABACABADABACABA FABACABADABACABAEABACABADABACABA	_____
8	ABACABADABACABAEABACABADABACABAFABACABADABACABAEABACABADABACABA GABACABADABACABAEABACABADABACABAFABACABADABACABAEABACABADABACABA HABACABADABACABAEABACABADABACABAFABACABADABACABAEABACABADABACABA GABACABADABACABAEABACABADABACABAFABACABADABACABAEABACABADABACABA	_____
9	(yikes!)	_____

Question 1: How many number patterns can you find on the chart? Can you continue the pattern without writing out all of the names?

USE column **A** to answer these questions:

2. What is the number of the first genie to have over 1000 letters in his/her name?

3. What is number of the first genie to have over one million letters in his/her name?

4. How many letters are in the name of the 26th genie?

Use column **B** to answer these questions:

5. It takes about 2 seconds to say "Abacaba-Dabacaba." The time to say a genie name doubles with each genie...

Genie #	approximate # of seconds to say name
4	2
5	4
6	8
7	16

... and so on. See column **B**.

How many seconds would it take to say the name of the 26th genie (nonstop)? ____________

How many minutes is this? (÷ 60) ____________

How many hours is this? (÷ 60) ____________

How many days is this? (÷ 24) ____________

Genie #	column **A** # of letters	column **B** time in seconds
1	1	
2	3	
3	7	1
4	15	2
5	31	4
6	________	________
7	________	________
8	________	________
9	________	________
10	________	________
11	________	________
12	________	________
13	________	________
14	________	________
15	________	________
16	________	________
17	________	________
18	________	________
19	________	________
20	________	________
21	________	________
22	________	________
23	________	________
24	________	________
25	________	________
26	________	________

6. Do you think it's possible to actually say the name of the 26th genie? Why or why not?

Teacher Guide to MAGGIE AND THE ABACABA GENIES – PATTERNS 2

This activity reveals some additional properties of the pattern, helps students explore sums of powers of 2, and encourages conjecture-making and algebraic reasoning.

Prerequisite: These activities use the terminology *"Powers of 2."* Before starting this activity, explain to your students that a power of two is any number you can get starting with 1 and doubling. 1, 2, 4, 8, 16, 32, 64, ... etc. are all powers of 2. (More accurately, a power of two is any number you can get starting with 1 and doubling or halving, so 1/2, 1/4, etc. are also powers of 2. This ties in to negative exponents, which your students may or may not be ready for.)

Answer Key

1. **Every Others**

This is a clever method to generate ABACABA patterns. Note that this method can be extended to any number of letters.

2. **Letter Count**

The completed chart is shown here. Patterns include: every column contains powers of 2, and every row contains powers of 2. Note the connection to the first problem!

Genie #	Name	# of As	# of Bs	#of Cs	# of Ds	# of Es	# of Fs
1	A	1	0	0	0	0	0
2	ABA	2	1	0	0	0	0
3	ABACABA	4	2	1	0	0	0
4	ABACABADABACABA	8	4	2	1	0	0
5	ABACABADABACABAEABACABADABACABA	16	8	4	2	1	0
6	ABACABADABACABAEABACABADABACABAFABACABADABACABAEABACABADABACABA	32	16	8	4	2	1

3. **Patterns of Powers of Twos**

(a) Find these sums. Be on the look-out for patterns and short-cuts!

1 + 2 + 4 = __7__ 1 + 2 + 4 + 8 = __15_ 1 + 2 + 4 + 8 + 16 = __31_

1 + 2 + 4 + 8 + 16 + 32 = __63_ 1 + 2 + 4 + 8 + 16 + 32 + 64 = _127_

1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + 1024 = _2047_

(b) Describe a shortcut to find the sum of a series of powers of 2.

The sum is one less than the next power of two...
Double the last number in the series and subtract one...

Name ___________________________

MAGGIE AND THE ABACABA GENIES – PATTERNS 2

1. **Every Others**

Below are 15 blanks.
(a) Write an A in every other blank, starting with an A in the first blank.
(b) Write a B in every other empty blank, starting with a B in the first available blank.
(c) Write a C in every other empty blank, starting with a C in the first available blank.
(d) Write a D in every other empty blank, starting with a D in the first available blank
(*NOTE: there is really only one available blank at this point.)

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

If you did this correctly, you should have written the name of a genie!

2. **Letter Count**

(a) Complete the following chart.

Genie #	Name	# of As	# of Bs	#of Cs	# of Ds	# of Es	# of Fs
1	A	1	0	0	0	0	0
2	ABA	___	___	___	___	___	___
3	ABACABA	___	___	___	___	___	___
4	ABACABADABACABA	___	___	___	___	___	___
5	ABACABADABACABAEABACABADABACABA	___	___	___	___	___	___
6	ABACABADABACABAEABACABADABACABAFABACABADABACABAEABACABADABACABA	___	___	___	___	___	___

(b) Describe any patterns you find:

3. **Patterns of Powers of Twos**

(a) Find these sums. Be on the look-out for patterns and short-cuts!

1 + 2 + 4 = _____ 1 + 2 + 4 + 8 = _____ 1 + 2 + 4 + 8 + 16 = _____

1 + 2 + 4 + 8 + 16 + 32 = _____ 1 + 2 + 4 + 8 + 16 + 32 + 64 = _____

1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + 1024 = _____

(b) Describe a shortcut to find the sum of a series of powers of 2.

FRACTALS, TREES, and ABACABA rulers

"Abacabadabacaba..." can be thought of as a fractal word. A "fractal" is a self-similar figure, meaning that if you zoom in on the figure, you will see the same pattern repeating over and over again. Fractals are not only fun and captivating, they can be used to explore number, patterns, and fractions, and are suitable for grades 3 and up.

Teacher Notes for FRACTAL TREE

Give your students copies of the Fractal Tree worksheet and guide them in creating a fractal figure. Tell them: *"A fractal is a special kind of shape that repeats itself over and over again at smaller and smaller scales, so that if you were to look at a tiny of the shape, it would look exactly like the whole figure. They are made by following a simple rule over and over again, and they make a shape that infinitely complicated. Crazy? Let's make one!"*

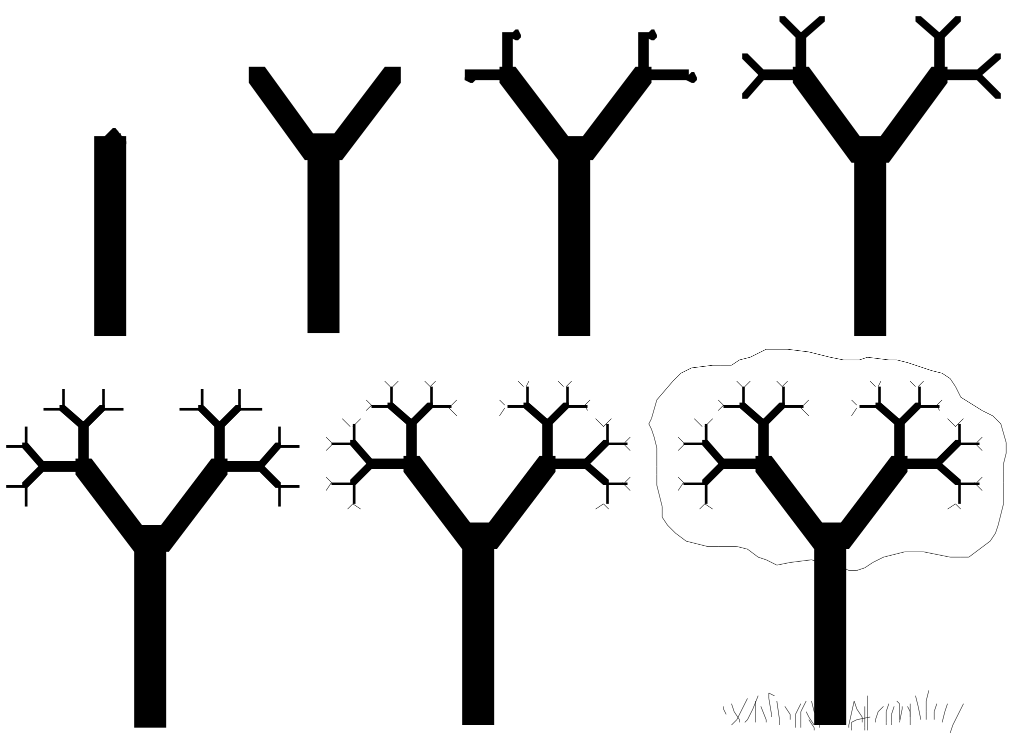
Step 1: Draw a tree trunk on the lower half of the page. Call the length of the trunk "1." Fill in the first row of the chart: 1, 1, and 1.

Step 2: Draw two branches coming up from the trunk, each half the size of the trunk. Fill in the second row of the chart: 2, 1/2, and 1.

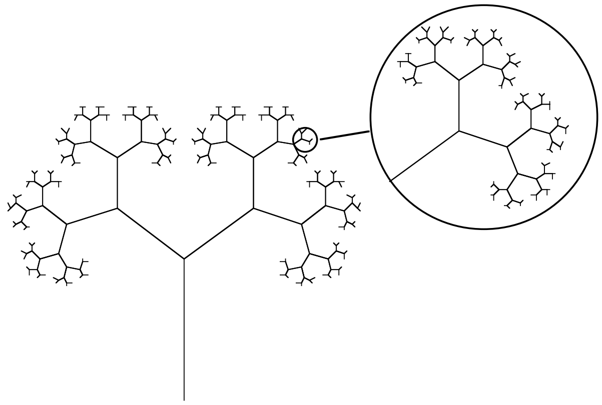
Step 3: Draw two branches coming up from the end of each branch, each half the size of the branches. Fill in the third row of the chart: 4, 1/4, and 1.

Steps 4+: Continue to add two smaller half-size branches onto the end of each branch, and continue filling in the chart. Keep going until the branches are too small to make any more.

Discuss the shape and the patterns. The number of branches keeps doubling, the size keeps halving, but the total length added remains 1 every step. If you could continue infinitely, the tree would have *inifinite length*, but it wouldn't grow off the page!

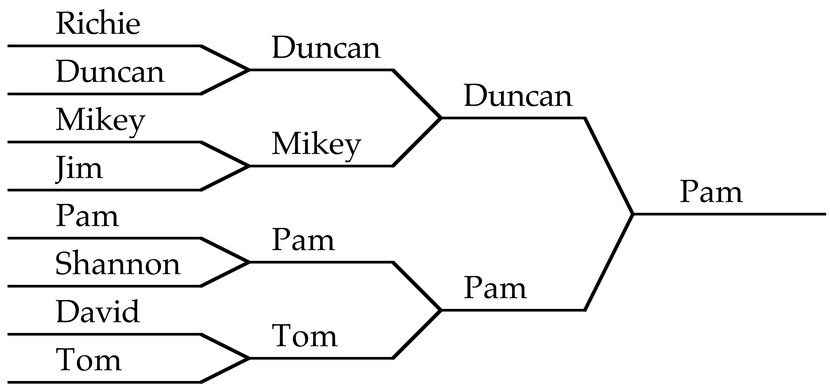


Teacher Notes: FRACTAL TREE continued



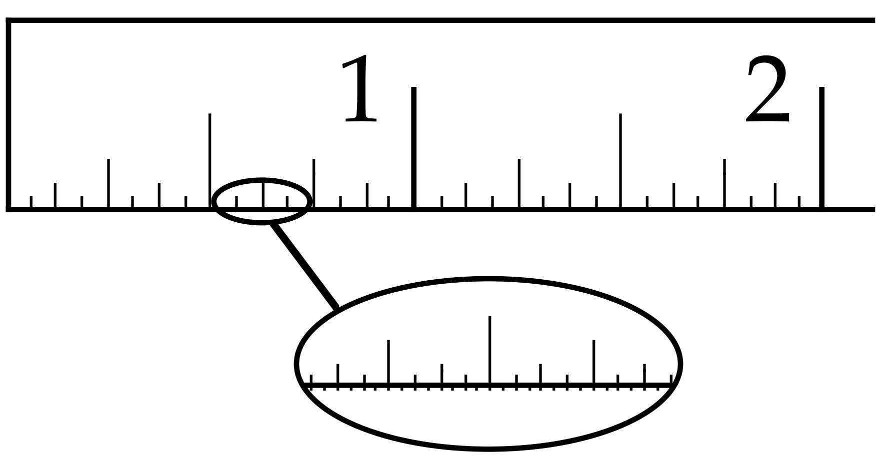
It is not hard to imagine that the branching and doubling could be continued infinitely, and if we could only magnify our view enough times, we would see the same patterns continuing forever and ever...

This same pattern is followed (in reverse) in a play-off schedule where teams or players are paired off with the winner of each round progressing to the next while the loser is eliminated:



If we move from the top to the bottom of the playoff tree, we'll notice that the name closest to the top of the chart appears in column 1, the next highest name is in column 2, then column 1, then 3, then back to 1, and so on. The pattern goes: 1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1 – this is the ABACABA pattern again!

Can your students spot the same pattern again on an English ruler? Each unit is divided into halves by a long mark, those halves are each divided into quarters by smaller marks, and so on.



If the shortest marks are A, the next length B, and so on, the pattern of the marks is the ABACABA pattern! WOW!

Name ____________________________

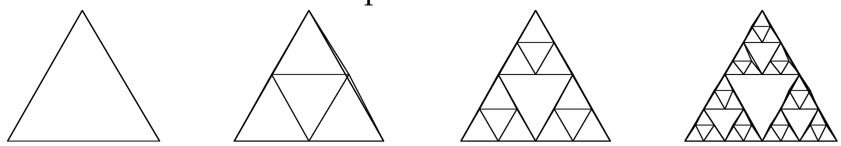
Tree Fractal

Step	# of new branches	length of a new branch	total length added this step
1	________	________	________
2	________	________	________
3	________	________	________
4	________	________	________
5	________	________	________
6	________	________	________

FRACTALS 2 – Sierpinski

Waclaw Sierpinski (1882-1969) was a Polish mathematician who is best known for his work with fractals and space-filling curves.

The Sierpinski Triangle is one of the most interesting and beautiful fractals. Two worksheets are given here: the first (label v.1 in the upper left corner) is appropriate for grades 3-4, the second one titled "Sierpinsky Triangle ...and area" (v.2) is appropriate for students 4th grade and higher who have had experience with fractions and area concepts.



Guide your students with the following directions:

1. Find the midpoint of each side of the triangle.
2. Connect the midpoints to each other with straight lines – follow the dots on the worksheet.
3. Lightly shade in this center triangle. Tell your students this triangle has been "removed" and it is now a "hole."
4. Complete the data for the first row of the chart.
5. Now there are 3 triangles remaining – repeat these steps, that is, find the midpoints of each of the triangles, connect them, and shade in the middle portions of the triangles, and complete the next row of the chart.
6. Repeat on the 9 remaining triangles, and continue until the triangles are too small to work with. Most student will probably not be able to draw beyond the fourth step – this is OK.

Answer Key

Version 1 (grades 3-4)

Sierpinski Triangle

Step	# new Δ "holes"	Δ side length
1	1	8
2	3	4
3	9	2
4	27	1
5	81	1/2
6	243	1/4

Version 2 (grades 4-8)

Sierpinski Triangle . . . and area

Step	# new Δ holes	area of new hole	remaining area
1	1	1/4	3/4
2	3	1/16	9/16
3	9	1/64	27/64
4	27	1/256	81/256
5	81	1/1024	243/1024

With older children, have them re-compute the remaining areas as decimals and make a prediction about what happens to the area if the steps were continued infinitely. Answer: the area goes to zero!

Abacaba Connections

Call the smallest triangular "holes" A, the next largest B, the next largest C, and so on. Follow the edge of the triangle and notice that the vertices of different-sized triangular holes touch the edge. What is the order in which they touch the edge? ABACABADABACABA...!

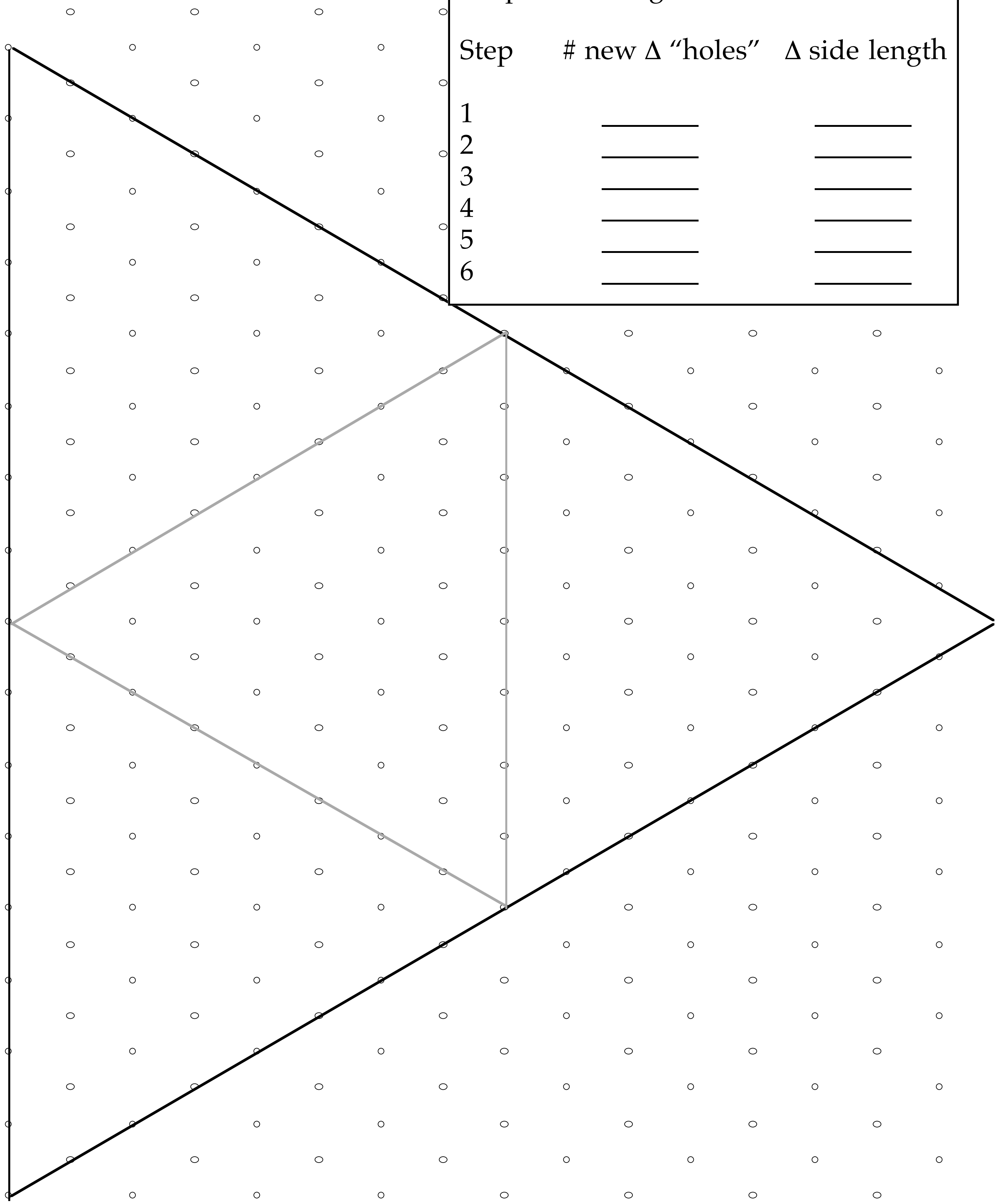
The pattern continues no matter which line segment you follow anywhere in any triangle within the fractal.

v.1

Name ______________________________

Sierpinski Triangle

Step	# new Δ "holes"	Δ side length
1	________	________
2	________	________
3	________	________
4	________	________
5	________	________
6	________	________

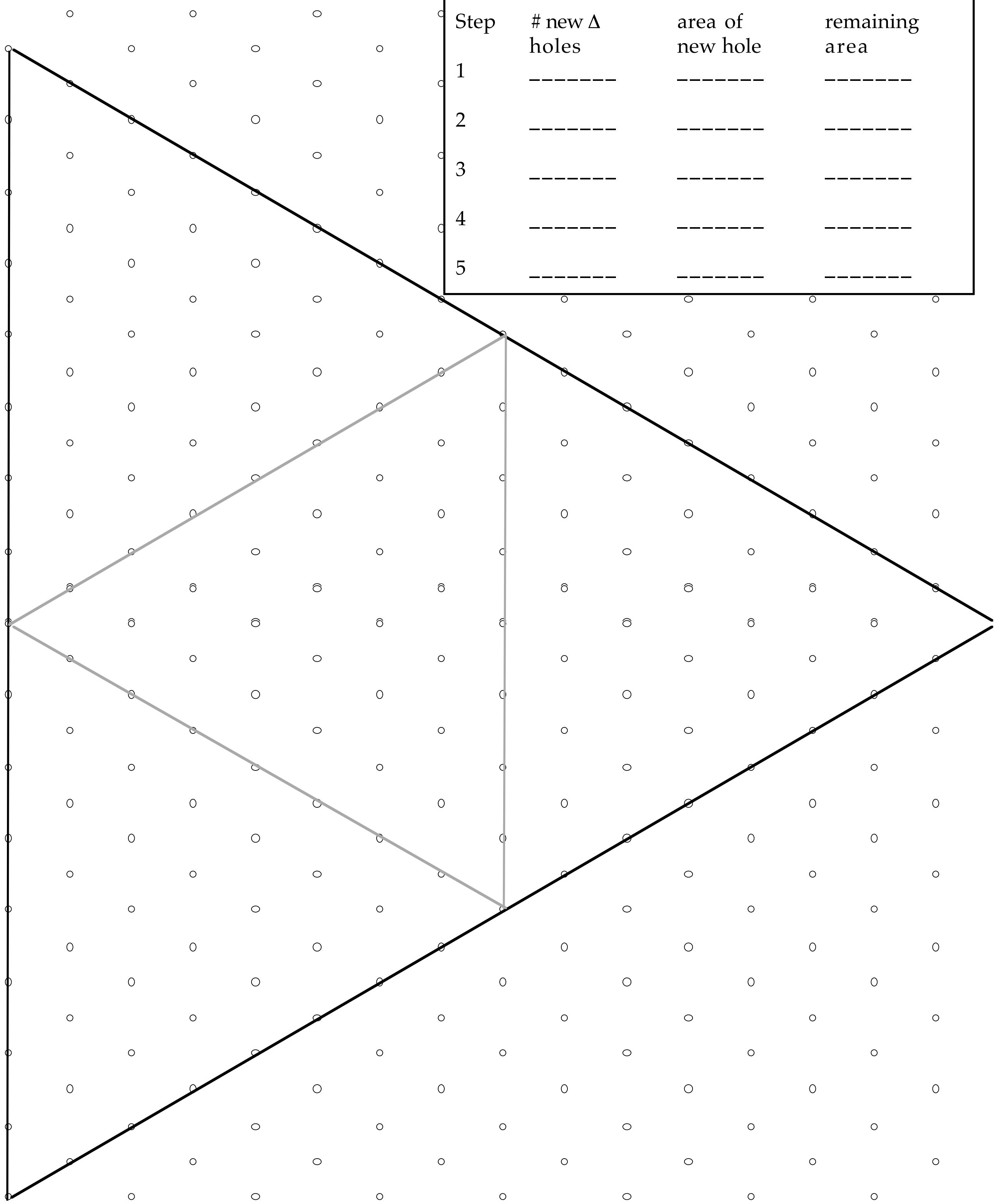


v.2

Name ______________________________

Sierpinski Triangle . . . and area

Step	# new Δ holes	area of new hole	remaining area
1	_______	_______	_______
2	_______	_______	_______
3	_______	_______	_______
4	_______	_______	_______
5	_______	_______	_______



Pascal's Triangle and Abacaba

This arithmetic triangle was studied in-depth by Blaise Pascal (1623-1662), a French mathematician. The triangle is constructed with a 1 at the upper vertex and 1s down the left and right sides. Numbers on the interior of the triangle are the sum of the two numbers above it.

Pascal's triangle is filled with fascinating patterns. The triangle on the following page has 17 rows, which is enough to find some very rich and interesting patterns. Individual copies for each of your students are a must for pattern-seeking, and a big copy of the triangle makes an excellent poster for the classroom – your students will continue to find patterns all year long.

A web search on "Pascal's Triangle" will produce thousands of links to interesting patterns. Here's a couple of patterns that relate to Abacaba.

An "Odd" Connection

Every row starts with a 1, so there is at least one odd number in every row. Ask your students to count how many consecutive odd numbers (how many in a row) begin each of the lines.

The pattern is: 1, 2, 1, 4, 1, 2, 1, 8, 1, 2, 1, 4, 1, 2, 1, 16, 1, ... It is the ABACABA pattern!

Even "Odder"

For a big surprise, have your students circle and color the odd numbers in Pascal's triangle. The result? A Sierpinski Triangle! Wow!

Name ___________________________

Pascal's Triangle – named for Blaise Pascal (1623-1662)

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

1 11 55 165 330 462 462 330 165 55 11 1

1 12 66 220 495 792 924 792 495 220 66 12 1

1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1

1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1

1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1

1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1

1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1

POP-UP FRACTALS

These eye-popping fractals are a big hit with kids. It's easy to create something complicated and beautiful using ideas of fractals. The following page contains the pattern for making this work of art.

Supplies:

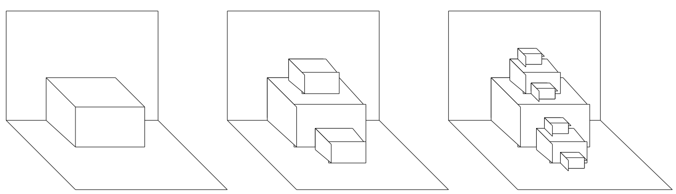
Copies of the handout on the next page, Scissors
optional: 9" x 12" construction paper and glue sticks

Have students fold the page in half along the dotted line. Fold so that the print is on the <u>outside</u> of the folded page.

Cut along the two heavy line segments that connect to the crease.
Open the page and reverse the fold in the center flap and refold the page. The flap should now be completely (and neatly!) inside the folded paper.

Two move heavy lines are now seen to reach the creases in the center of the paper. Cut along these also, then reverse the two flaps as above. Continue in this manner.

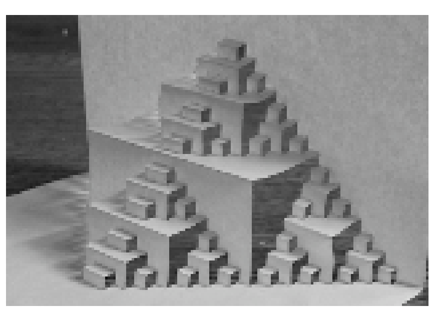
Point out that: The first cuts made one flap, the second cut made two flaps, and the third made 4. How many flaps are made by the final cut? Discuss: How is this a fractal?

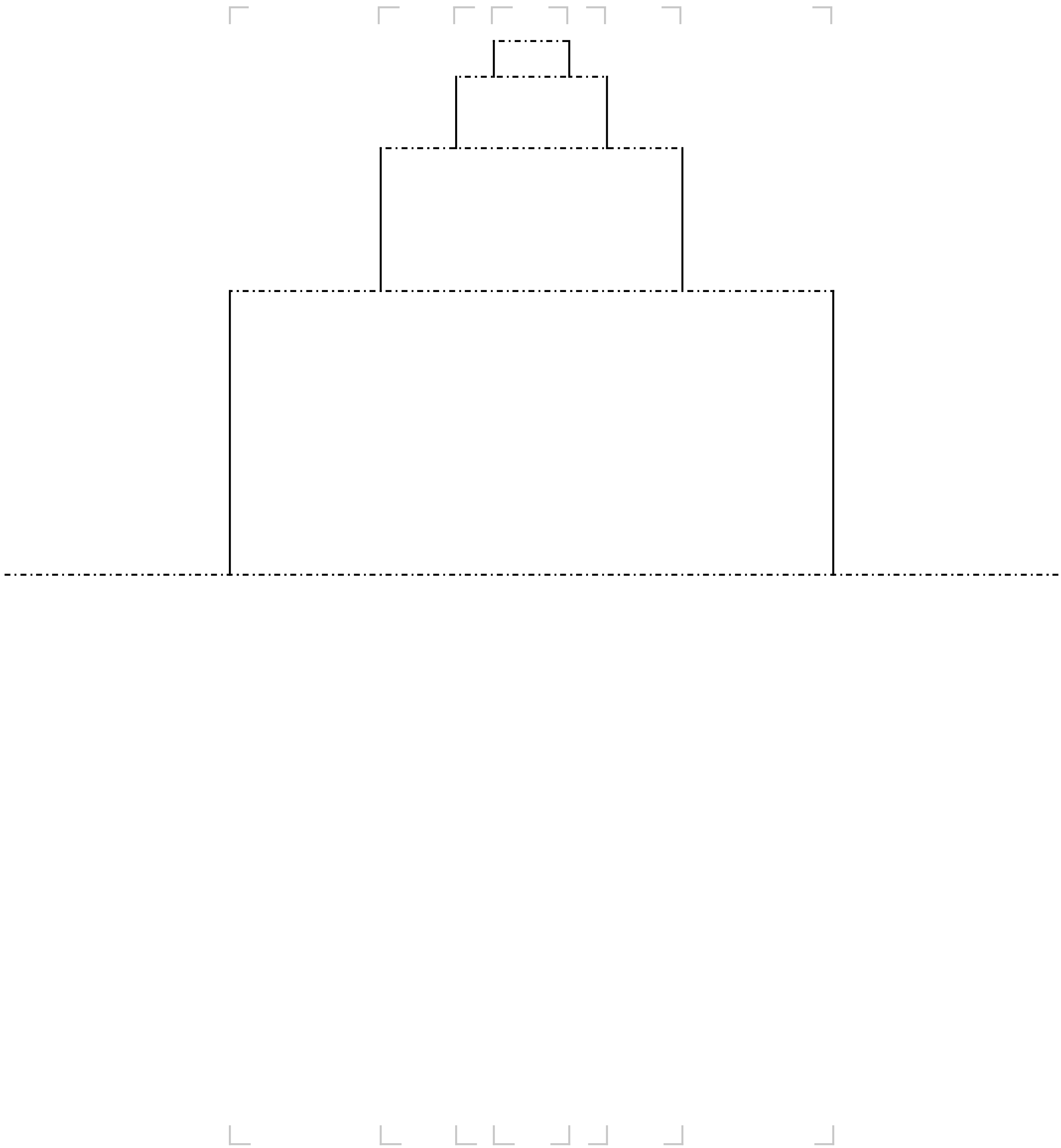


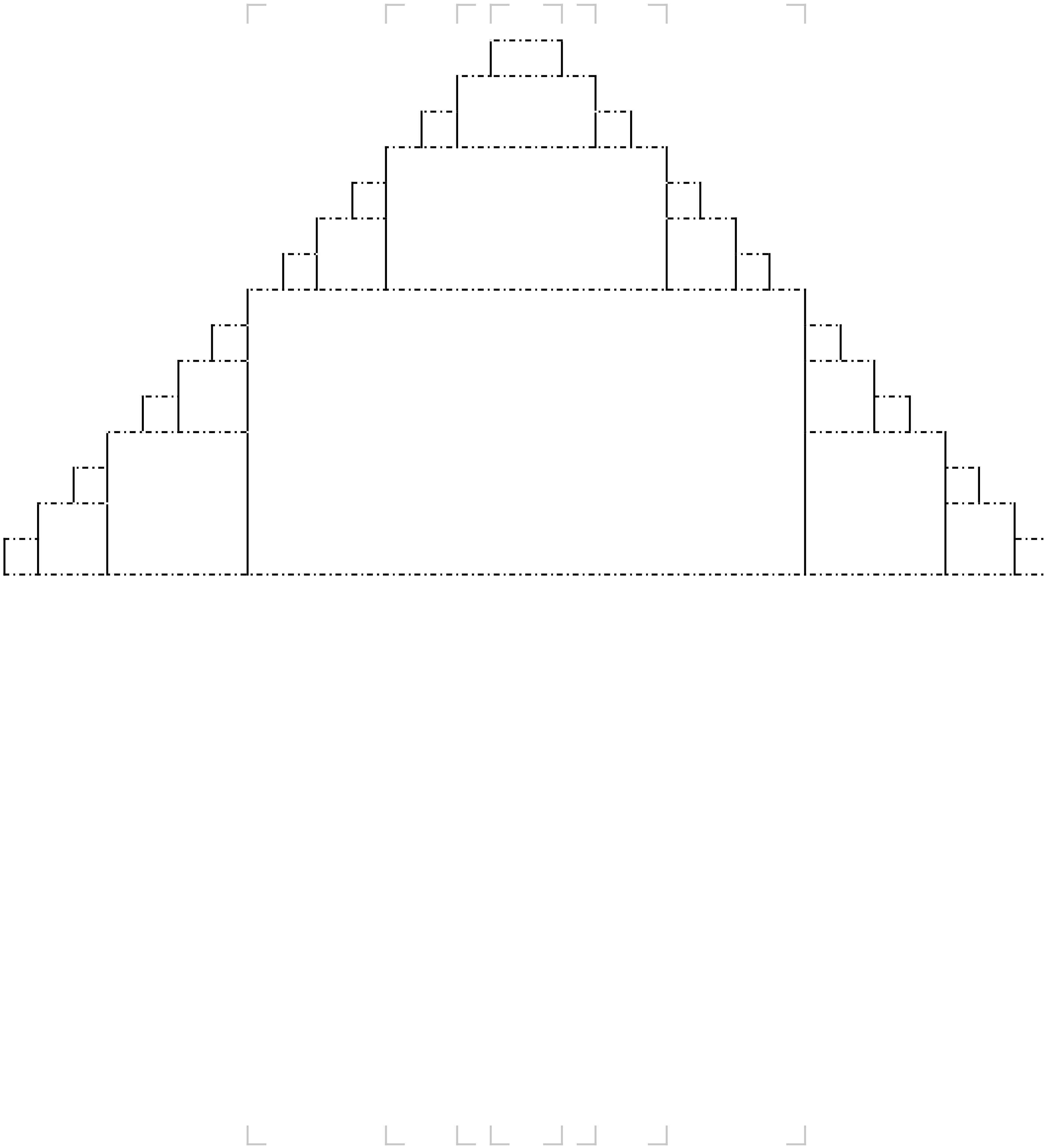
When it is done, you will have a beautiful 3-D pop-up form. This can be glued to a folded sheet of construction paper to make a lovely piece of art or an eye-popping greeting card. Have your students find other properties of their fractal to investigate!

ABACABA CONNECTION: If the smallest block is A, the next biggest is B, the next biggest C, and the largest central block is D, then as you "walk up" the stairs, you spell ABACABADABACABA!

POP-UP FRACTAL 2. The second template creates a more complicated fractal, shown below. Some of your students may want to try this! Show them the picture and have them figure out how to make it. (A picture of this is hidden in "Maggie and the Abacaba Genies."





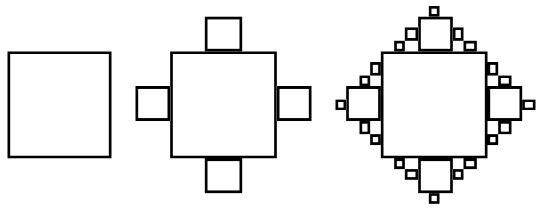


Fractal Art

For this assignment, you will create your own original fractals.

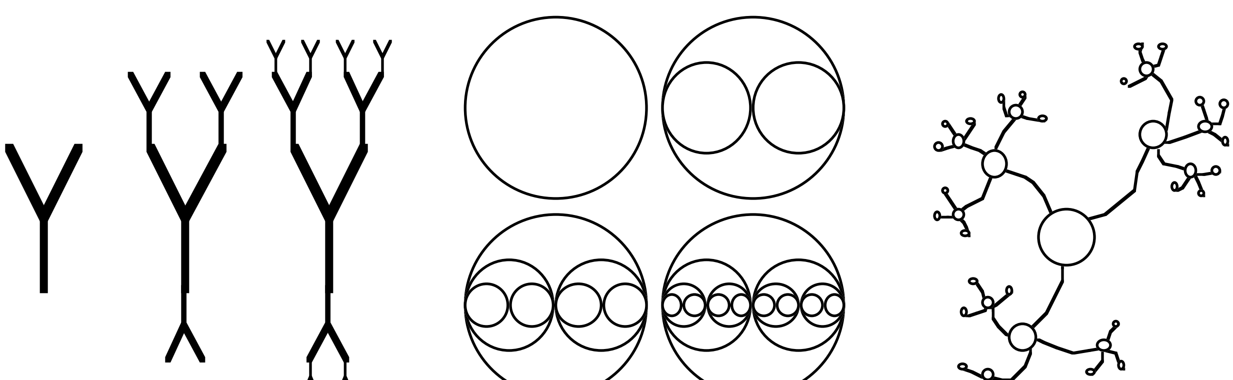
1. Choose a base shape and a rule.
2. Apply your rule on the shapes until you have smaller copies of the base shape.
3. Repeat step 2 many times.

A good rule must result in smaller copies of the base shape so that it may be repeated on the smaller copies. A rule may involve adding or subtracting parts of the figure, using other shapes, using color, or any other creative idea you can think of. You should be able to zoom in on your figure and keep seeing the same shapes in the same proportions.



Square Fractal

Here's some examples. Above, the base shape is a square, the rule is to add squares around the square next to the center third of each side. In the first fractal below, the base shape is the letter "Y." Add two y's standing on top of the big Y, and add one small y upside-down on the bottom. The rule is repeated on all unconnected ends of the figure. The next fractal has a base shape as a circle with the rule "add two circles next to each other in each circle. Can you determine the base shape and the rule for the Alien Fractal shown below?



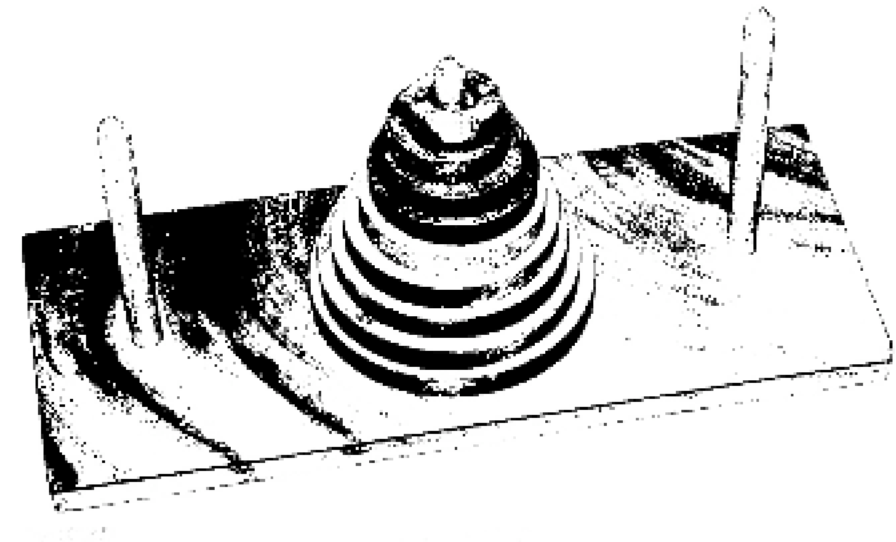
Y Fractal Circle Fractal Alien Fractal

PROBLEM SOLVING: Roly-Annie and the Towers of Hanoi

Teachers Guide

The following page contains a wonderful problem-of-the-week suitable for middle schoolers. It is a playing card version of a puzzle known as "The Towers of Hanoi."

The original Towers of Hanoi puzzle (shown below) consists of a stack of disks which can be moved one at a time onto one of three pegs. The oject is to move the entire stack from one peg to another without ever placing a larger disk onto a smaller disk. In the "Roly-Annie" version on the following page, the object is to move a stack of cards without ever placing a greater number on a lesser number.



Amazingly, the order that the cards (or disks) must be moved is 1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1, 5... It is the ABACABA pattern once again!

ANSWER KEY

1. Minimum moves with 7 cards = 127.

2. 1213121412131215121312141213121б...

3. Patterns: those shown above, plus:
   - odd numbers cards move in all the same direction (clockwise or counterclockwise)
   - even numbered cards move in the opposite direction of the odd number cards

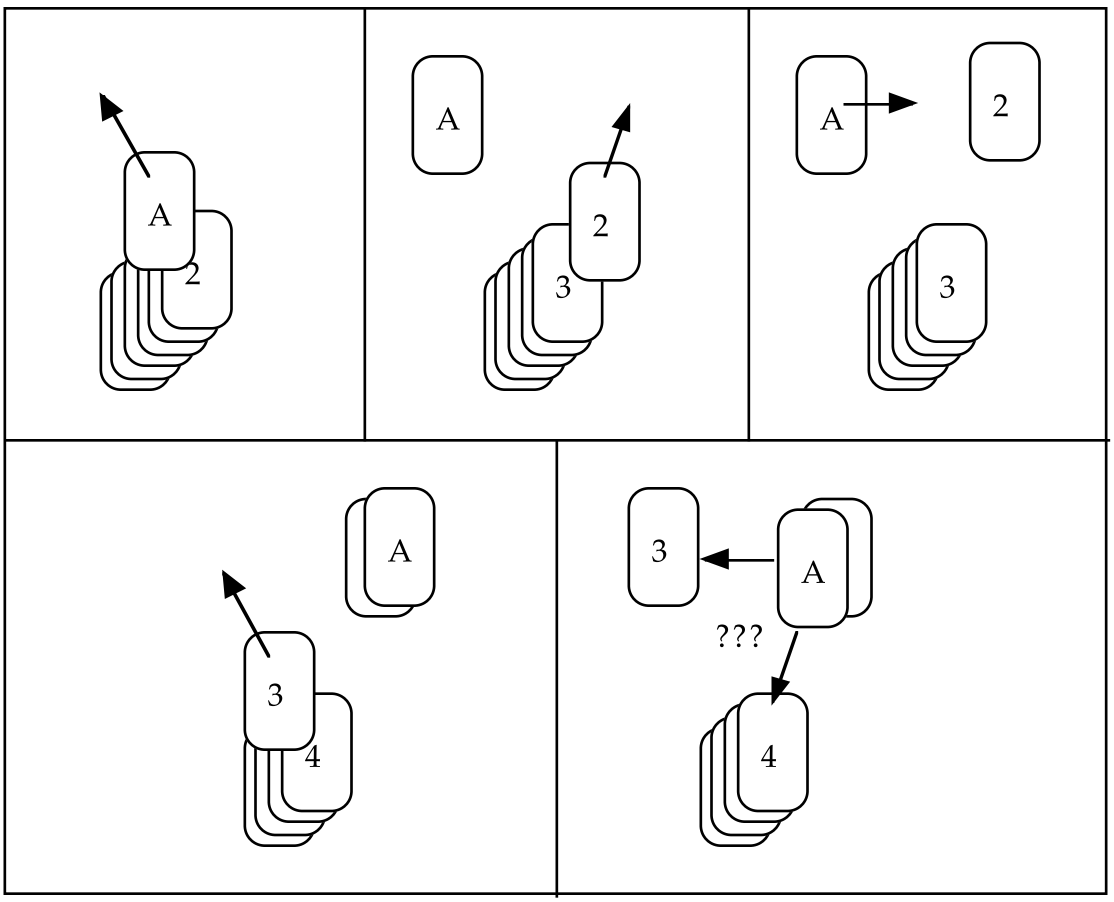
4. With 26 cards, the number of moves is equal to the number of letters in the name of the 26th genie: 67,108,863 moves. At one move per second, the game would last about 777 days, or over 2 years of non-stop play!

Roly-Annie

"Roly-Annie" is a solitaire card game named after the card queen of Mississippi riverboats, "Roly" Annie Keim. To play, use the cards ace through seven. Make the odd numbered cards black and the even numbered cards red, and stack them in order so that when the stack is face up, the ace is on top.

The cards start in the middle stack. You are allowed to make two additional stacks in this game, a right stack and a left stack. You're allowed to move **the top card** of any stack (on the table or in your hand) to the top of any other stack (on the table or in your hand). However, you may move **only one card** at a time, and you may never place a higher numbered card on top of a lower numbered card; you may **only place a lower number on a higher number** (the ace has a value of 1). You may play any card on an empty stack.

The **object** of the game is get all of the cards into either the left or right stack. The start of a game is shown below.



1. What is the minimum number of moves required to win this game?

2. List at least the first 20 moves. (You may wish to devise your own notation.)

3. Describe any patterns you found while investigating this game. Describe a strategy that will allow you to win every time without making a mistake.

4. You and a friend decide to race. You split a deck of cards and number the cards in each half from 1 to 26. Playing by the same rules as before, about how long would it take to decide a winner?

ADDITONAL EXTENSIONS & CONNECTIONS

Number Systems

The binary number system is the simplest and most significant of all positional number systems.

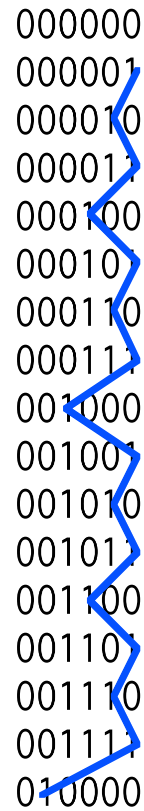
In binary, only two digits are used, usually zero and one. The value of a written number is determined by the arrangement of zeros and ones, with the value of each place being a power of 2 rather than a power of 10 as in our familiar system. The number 19, for example, would be written as follows:

19 (base 10)	=	**0**	**1**	**0**	**0**	**1**	**1**	digits of binary #
		32	*16*	*8*	*4*	*2*	*1*	
		2^5	2^4	2^3	2^2	2^1	2^0	

$$19 = 2^4 + 2^1 + 2^0$$

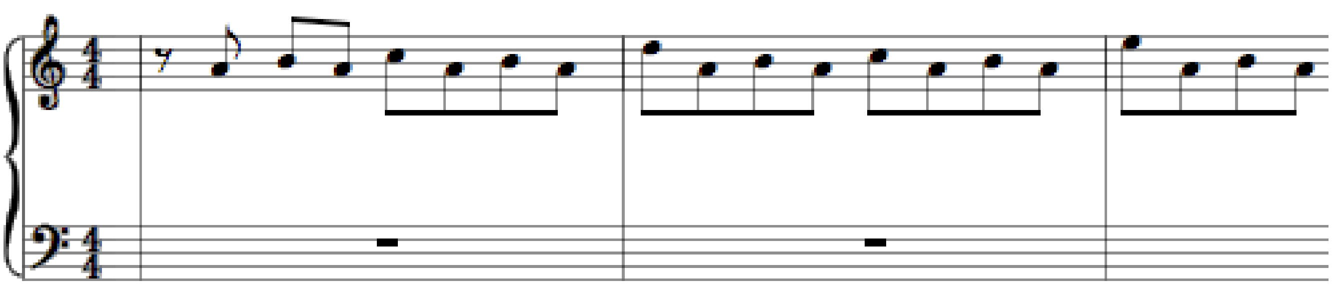
The count from 0 to 8 in binary is: 0, 1, 10, 11, 100, 101, 110, 111, 1000

The abacabadabacaba pattern is repeated at infinite levels in this simple counting pattern. The start of one such pattern is shown to the right. Can you find others?



Music

Abacabadabacaba sounds like it could represent a series of notes. Indeed, if abacabadabacaba is played on piano, the result is beautiful and haunting. A CD with the simple melody and a much more complicated, fully-orchestrated version is available at **www.lulu.com/mnaylor,** or download it for free from **www.abacabadabacaba.net**



Philosophy and Poetry

It's fun to think about how every decision we make leads us in a new direction, as if our lives are an infinite fractal tree. Here's a poem that reflects those decisions... it has the structure *abacabadabacaba*. Maybe your students could write a poem with this structure!

Decision Tree

by Michael Naylor

And keep my conscience clear and bright.
I'll do what I know is right
Next time, who knows? I just might!
"I shouldn't do it," so I thought.
I guess I'm doomed to live this way.
Now the chance has slipped away
Maybe on some other day!
Should I do it? Should I not?
It wasn't worth it, I would say.
Now I'm full of guilty thoughts
But that's a tiny price to pay!
I went and did it anyway.
Tomorrow I will stay away.
I'll just hope I don't get caught
They didn't catch me yesterday!

Naylor, M. "Tree Diagram," *College Math Journal*, 32, 3 (May 2001).

Additonal Books and CDs by Michael Naylor

"Abacaba Illustrated" contains over 1 million letters of the Abacaba pattern, interspersed with 30 illustrations that connect in some way to the pattern.

"Abacaba Unabridged" is for the serious Abacaba collector. 4 volumes and over 1600 pages of very small print are needed to capture the entire 67 million letters of the complete Abacaba pattern, "all the way to Z" (and back)!

"Abacaba Music" is a CD with music based on the Abacaba pattern.

These are available at **www.lulu.com/mnaylor**